BLACK N WHITE

Learn Today Lead Tomorrow

jag....

NAME	
ROLL NUMBER	
SEMESTER	1st
Batch	7th
COURSE CODE	DCA1103
COURSE NAME	**BASIC MATHEMATICS**

SET - I

Q.1.a) By using truth tables, check whether the propositions $\sim(p \wedge q)$ and $(\sim p) \vee (\sim q)$ are logically equivalent or not?

Solution .:-

Logical Equivalence

Two propositions are logically equivalent if they have the same truth value for all possible assignments of truth values to their component statements (p and q in this case).

Truth Tables

We can construct a truth table to evaluate the truth values of both propositions for all combinations of truth values for p and q.

p	q	p ↔ q	(p → q) ∧ (q → p)
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Analysis

As you can see from the truth table, the propositions p ↔ q and (p → q) ∧ (q → p) have the same truth values in all four cases. This means they are logically equivalent.

Explanation

- **p ↔ q (p if and only if q):** This statement signifies that p and q must both be true or both be false for the entire proposition to be true.
- **(p → q) ∧ (q → p):** This conjunction expresses that if p is true, then q must also be true (p implies q), and conversely, if q is true, then p must also be true (q implies p).

Q.1.b) A Spaceship moves in a circular orbit of radius 7200 KM around the earth. How far does it travel while sweeping an angle of 100^0.

Solution .:- The formula to calculate the length of an arc in a circle is:

$$\text{Arc Length} = \frac{Angle\ x\ Radius}{360^0}$$

⇨ Given

- Radius *(r)* of the orbit = 7200 km
- Angle *(0)* swept by the spaceship = 100^0

We can plug these values into the formula :

$$\text{Arc Length} = \frac{100^0\ x\ 7200\ km}{360^0}$$

$$\text{Arc Length} = \frac{100\ x\ 7200\ km}{360}$$

$$\text{Arc Length} = \frac{720000\ km}{360}$$

Arc Length = 2000 km

➔So , the spaceship travels 200 km while sweeping an angle of 100^0

Q.2.a) Find the nth term of the sequence $3,\ 2,\ \frac{5}{3},\ \frac{6}{4},\ \frac{7}{5},\ldots$ and find its limit if exist.

Solution .:- To find the nth term of the sequence, let's observe the pattern:

1. The numerator starts at 3 and increases by 1 for each subsequent term.
2. The denominator starts at 1 and increases by 1 for each subsequent term.

⇨ So, the nth term of the sequence can be written as $\frac{n+2}{n}$.

Now, to find the limit of the sequence if it exists, we can analyze the behavior of the sequence as n approaches infinity.

$$\lim n \to \infty \frac{n+2}{n}.$$

Using limit properties, we can divide both the numerator and the denominator by n:

$$\lim n \to \infty \frac{1+\frac{2}{n}}{n}.$$

As *n* approaches infinity , $\frac{2}{n}$ approaches 0. So, the limit becomes :

$$\lim n \to \infty \frac{1+0}{1} = 1.$$

➔Therefore, the limit of the sequence is 1.

Q.2.b) Find the value of $\left(-1+i\sqrt{3}\right)^{10}$ using De Moivre's theorem .

Solution .:- The polar form of a complex number $z =$ $+bi\ is\ given\ by\ r\ (\cos(0) +$ $i\sin(0)), where\ r\ is\ the\ magnitude\ of\ z\ and\ 0\ is\ the\ argument\ of\ z$.

Here , $a = -1, b = \sqrt{3}, r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{1+3} = \sqrt{4} = 2\ , and\ 0 =$ $\arctan\left(\frac{a}{b}\right) = \arctan\left(\frac{\sqrt{3}}{-1}\right) = \arctan\left(-\sqrt{3}\ \right)$.

Now , let's find 0 in the interval [0 , 2π) :

$0\ 0 = \frac{\pi}{3} = \frac{2\pi}{3}$

So , the polar form of $-1 + i\ \sqrt{3}\ is\ 2\ (cos\left(\frac{2\pi}{3}\right) + isin\left(\frac{2\pi}{3}\right))$

Now, applying De Moivre's theorem, we raise this to the power of 10:

$(2(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)))\text{^}10$

By De Moivre's theorem, this becomes:

$2^{10}(\cos\left(10.\frac{2\pi}{3}\right) + i\sin\left(10.\frac{2\pi}{3}\right))$

Now , $\frac{20}{3}\ is\ equivalent\ to \frac{6\pi}{3} + \frac{3\pi}{3} = 2\pi +$ $\frac{2\pi}{3}, where\ is\ the\ same\ position\ as\ \frac{2\pi}{3}\ on\ the\ unit\ circle.$

So , $\cos\left(\frac{20\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)\ and \sin\left(\frac{20\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right)$.

$= 1024(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right))$

$= 1024\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= -512 + 512i\sqrt{3}$

➔So , $\left(-1+i\sqrt{3}\right)^{10} = -512 + 512\ i\sqrt{3}.$

Q.3) Apply Cramer's rule to solve the system of equations: $3x + y + 2z = 3$; $2x - 3y - z = -3$; $x + 2y + z = 4$.

Solution .:-

let's denote the coefficients of the variables

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

According to Cramer's rule, the value of each variable is given by the determinants of matrices formed by replacing each column of A with the column matrix B, divided by the determinant of A.

Now, let's compute the determinant of A :

$$|A| = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

Applying cofactor expansion along the first row, we have:-

$$|A| = 3\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

Evaluating the 2x2 determinants:

$|A| = 3 ((-3) (1) - (-1) (2)) - 1 ((2) (1) - (-1) (1) + 2 ((2) (2) - (-3) (1))$

$|A| = 3 (-5) - 1 (3) + 2 (7) = - 15 - 3 + 14 = - 4$

The determinant of matrix A is - 4.

Next, we will compute the determinant of matrices obtained by replacing each column of A with B :

For the determinant of matrix A , we replace the first column of A with the column matrix B :

$$Ay = \begin{bmatrix} 3 & 3 & 2 \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{bmatrix}$$

Applying cofactor expansion along the second column, we have:

$$Ay = 3\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix} - (3)\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 3 & 3 \\ 1 & 4 \end{vmatrix}$$

Evaluating the 2x2 determinants:

$Ay = 3 ((3) (-1) - (2) (-1)) - (-3)((3)(1) - (2) (1)) + 2 ((3) (4) - (3) (1))$

$Ay = 3(-1) - (-3)(1) + 2(9) = -3 + 3 + 18 = 18$

The determinant of matrix Ay is 18.
Finally, for the determinant of matrixAz , we replace the third column of A with the column matrix B.

$$Az = \begin{bmatrix} 3 & 1 & 3 \\ 2 & -3 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

Applying cofactor expansion along the third column, we have:

$$Az = 3\begin{vmatrix} 3 & -3 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

Evaluating the 2x2 determinants:

Az= 3 ((2) (2) - (-3) (1)) – 1((2)(4) – (-3) (1)) + 3 ((2) (2) – (-3) (2))

Az=3(4+3)−1(8+3)+3(4+6)=21−11+30=40

The determinant of matrix Az is 40.

Finally, we can find the values of the variables using Cramer's rule:

$$\boldsymbol{x = \frac{Ax}{|A|} = \frac{-12}{-4} = 3 \quad y = \frac{Ay}{|A|} = \frac{-18}{-4} = \frac{9}{2} \quad z = \frac{Az}{|A|} = \frac{40}{-4} = -10}$$

SET - II

Q.4) One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be

(i) a diamond

(ii) not a black card

(iii) a black card (i.e., a club or, a spade)

(iv) not a diamond.

Solution .:-

(i) Probability of drawing a diamond:

There are 13 diamonds in a standard deck of 52 cards. So, the probability of drawing a diamond is:

P(diamond) = $\frac{Number\ of\ diamond}{Number\ of\ Cards} = \frac{13}{52} = \frac{1}{4}$

(ii) Probability of not drawing a black card:

There are 26 red cards in a standard deck of 52 cards (13 hearts and 13 diamonds). So, the probability of not drawing a black card (drawing a red card) is:

P(not black) = $\frac{Number\ of\ red\ cards}{Number\ of\ Cards} = \frac{26}{52} = \frac{1}{2}$

(iii) Probability of drawing a black card:

There are 26 black cards in a standard deck of 52 cards (13 clubs and 13 spades). So, the probability of drawing a black card is:

P(black) = $\frac{Number\ of\ black\ cards}{Number\ of\ Cards} = \frac{26}{52} = \frac{1}{2}$

(iv) Probability of not drawing a diamond:

There are 39 non-diamond cards in a standard deck of 52 cards (26 red cards and 13 black cards that are not diamonds). So, the probability of not drawing a diamond is:

P(not diamond) = $\frac{Number\ of\ non\text{-}\mathrm{diamon}\ \ cards}{Number\ of\ Cards}$ = $\frac{39}{52}$ = $\frac{3}{4}$

Q.5) In an office there are 84 employees. Their salaries are as given below:

Salary (Rs.)	2430	2590	2870	3390	4720	5160
Employees	4	28	31	16	3	2

i) Find the mean salary of the employees

ii) What is the total salary of the employees?

Solution .:-

Mean = $\frac{Total\ Salary}{Total\ Number\ of\ Employees}$

First, let's calculate the total salary by multiplying each salary by the number of employees who earn that salary, and then summing up these values:

Total Salary=(2430×4)+(2590×28)+(2870×31)+(3390×16)+(4720×3)+(5160×2)

Total Salary=9720+72420+88970+54240+14160+10320

Total Salary = 249830Rs.

Now, to find the mean salary, divide the total salary by the total number of employees, which is 84:

Mean Salary= $\frac{249830}{84}$

Mean Salary ≈ $2973.45\ Rs.$

So, the mean salary of the employees is approximately Rs. 2973.45.

Q.6) Consider 77, 73, 75, 70, 72, 76, 75, 72, 74, 76. Give standard deviation for the numbers given above.

Solution .:-

Standard Deviation $\sqrt{\frac{\sum_{i=1}^{n}(xi-\bar{x})^2}{n}}$

- ⇨ Where :-
  - xi *represents each individual number in the set.*
  - $\bar{x}$ *represents the mean of the numbers.*
  - n *represents the total number of values in the set.*

First, let's find the mean $\bar{x}$ of the number

$$\bar{x} = \frac{77 + 73 + 75 + 70 + 72 + 76 + 75 + 72 + 76}{10}$$

$$\bar{x} = \frac{740}{10} = 74$$

Now, let's calculate the sum of the squares of the differences between each number and the mean:

$$(77 - 74)^2 + (73 - 74)^2 + (75 - 74)^2 + (70 - 74)^2 + (72 - 74)^2 + (76 - 74)^2 + (75 - 74)^2 + (72 - 74)^2 + (74 - 74)^2 + (76 - 74)^2$$

=9+1+1+16+4+4+1+4+0+4

Now, divide the sum by the total number of values and take the square root:

Standard Deviation $= \sqrt{\frac{44}{10}}$

Standard Deviation $= \sqrt{4.4}$

Standard Deviation ≈ 2.0976

➔So, the standard deviation for the given set of numbers is approximately 2.0976.